Int J Theor Phys (2009) 48: 61–70
DOI 10.1007/s10773-008-9782-2

# Lowest Order Constrained Variational Calculation of Structure Properties of Protoneutron Star

**G.H. Bordbar · S.M. Zebarjad · R. Zahedinia**

Received: 16 April 2008 / Accepted: 16 June 2008 / Published online: 3 July 2008


**Abstract** We calculate the structure properties of protoneutron star such as equation of state, maximum mass, radius and temperature profile using the lowest order constrained variational method. We show that the mass and radius of protoneutron star decrease by decreasing both entropy and temperature. For the protoneutron star, it is shown that the temperature is nearly constant in the core and drops rapidly near the crust.

**Keywords** Protoneutron star · Structure · Maximum mass · Radius · Temperature profile

## 1 Introduction

Neutron star which is highly compact stellar objects is a result of supernova explosion. The structure properties of this object, especially its maximum mass, is of a great interest for astrophysicists. Since the compactness parameter for a neutron star is about 0.2–0.4, its structure should be studied using the general theory of relativity. In fact, the computation of the structure properties of a neutron star can be derived using the Tolman-Oppenheimer-Volkoff (TOV) equation [1].

Just after the supernova collapse, a newly-born neutron star called protoneutron star is formed. At these stages, the neutron stars are rich in leptons. This is due to the fact that the neutrinos are trapped in the protoneutron star matter. The temperature of protoneutron star is greater than 10 MeV [2–9]. Therefore, the high temperature of these stages cannot be neglected with respect to the Fermi temperature throughout the calculation of its structure. Depending on the total number of nucleons, a protoneutron star evolves either to black hole or to stable neutron star [3, 7–9]. Therefore, computation of the maximum mass of protoneutron star is of crucial importance.

G.H. Bordbar (✉) · S.M. Zebarjad · R. Zahedinia
Department of Physics, Shiraz University, Shiraz 71454, Iran
e-mail: bordbar@physics.susc.ac.ir

G.H. Bordbar · S.M. Zebarjad · R. Zahedinia
Research Institute for Astronomy and Astrophysics of Maragha, P.O. Box 55134-441, Maragha, Iran

In recent years, we have investigated the properties of neutron star matter [10–14]. In the present work, we intend to compute the structure properties of protoneutron star at different stages employing the lowest order constrained variational approach using the modern $AV_{18}$ potential [15].

## 2 Formalism

In this section, we give the formalism of calculation for the protoneutron star matter equation of state.

We write the total energy per baryon ($E$) as the sum of contributions from leptons and nucleons,

$$E = E_{lep} + E_{nucl}. \tag{1}$$

The contribution from the energy of leptons is

$$E_{lep} = \frac{m^4c^5}{\pi^2\rho\hbar^3}\int_0^\infty n(x)\sqrt{1+x^2}x^2dx, \tag{2}$$

where $\rho$ is the total number density of nucleons and $n(x)$ is the Fermi-Dirac distribution function, where $x$ defined,

$$x = \frac{\hbar k}{mc}\cdot \tag{3}$$

We calculate the contribution from the energy of nucleons using the lowest order constrained variational method. We consider up to the two-body term in the cluster expansion for the energy functional,

$$E_{nucl} = E_1 + E_2. \tag{4}$$

The one-body energy $E_1$ is

$$E_1 = \sum_{i=n,p}\frac{\hbar^2}{2m_i\rho\pi^2}\int_0^\infty n_i(k)k^4dk. \tag{5}$$

The two-body energy $E_2$ is

$$E_2 = (2A)^{-1}\sum_{ij}\langle ij|\left\{-\frac{\hbar^2}{2m}[f(12),[\nabla_{12}, f(12)]] + f(12)V(12)f(12)\right\}|ij\rangle_a, \tag{6}$$

where $f(12)$ and $V(12)$ are the two-body correlation function and nucleon-nucleon potential, respectively. $V(12)$ has the following general form [15],

$$V(12) = \sum_{p=1}^{18} V^p(r_{12})O_{12}^p. \tag{7}$$

Finally, we minimize the two-body energy, $E_2$, with respect to the variation in the two-body correlation function subjected to the normalization constraint. From this minimization, we get a set of Euler-Lagrange differential equations. The correlation functions are calculated by solving these differential equations and then, the two-body energy, $E_2$, is computed [16, 17]. This leads to the equations of state used in our present work.

## 3 Calculations of Protoneutron Star Structure Properties

In this section, we calculate the structure properties of neutron star in the different stages just after its formation by numerically integrating the TOV equation. The structure properties of neutron star in these stages such as maximum mass, radius and temperature profile are calculated using the modern microscopic equations of state derived from constrained variational method employing the Argonne $V_{18}$ potential [15] as the inter-nucleonic interaction. At low densities, we also consider a hot dense matter model and use the equations of state obtained by Gondek et al. [18] and Strobel et al. [19]. Here is our results for different stages.

### 3.1 Lepton Rich Protoneutron Star

Since a neutron star, at the beginning of its lifetime (protoneutron star), is opaque with respect to neutrinos therefore, it contains a high lepton fraction ($Y$), 0.3–0.4. Furthermore, at this stage of neutron star, the entropy per baryon ($s$) is nearly constant throughout the star, $1 - 2k_B$. Since, we are also dealing with uncharged neutron star matter, the lepton and proton fractions should be equal [2–5].

Our results for the pressure of protoneutron star matter as a function of mass density are presented in Fig. 1 at entropies $s = 1k_B$ and $s = 2k_B$ with $Y = 0.4$ and $Y = 0.3$. It is seen that for a fixed value of entropy, the pressure of protoneutron star matter increases by decreasing the lepton fraction. We see that for a given value of lepton fraction, the equation of state for $s = 2.0k_B$ is stiffer than for $s = 1.0k_B$. In Fig. 1, We have compared our results with those of Gondek et al. [18] and Strobel et al. [19]. We have seen that the results of Gondek et al. [18] are nearly in agreement with our results for $s = 2.0k_B$ and $Y = 0.3$. But, the equations of state calculated by Strobel et al. [19] are stiffer than those of ours.

In Fig. 2, we have presented the gravitational mass of protoneutron star as a function of central mass density for different values of entropy and lepton fraction. From Fig. 2, we can see that the gravitational mass increases by increasing the central mass density. It is shown that for different cases of entropy and lepton fraction, the gravitational mass exhibits different mass limits (maximum mass) as given in Table 1. From this table, the decreasing of maximum mass by decreasing the entropy and increasing the lepton fraction is seen. In

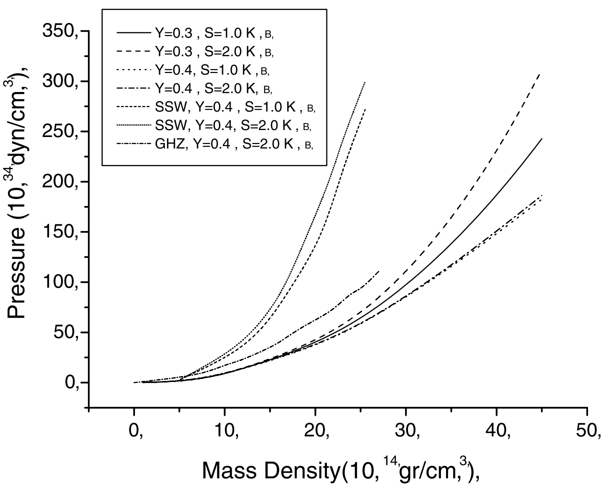



**Fig. 1** Our results for the pressure ($10^{34}$ dyn/cm$^2$) of the protoneutron star matter versus mass density ($10^{14}$ g/cm$^3$) at entropies $s = 1k_B$ and $s = 2k_B$ with lepton fractions $Y = 0.4$ and 0.3. The results of Gondek et al. [18] (GHZ) and Strobel et al. [19] (SSW) are also given for comparison

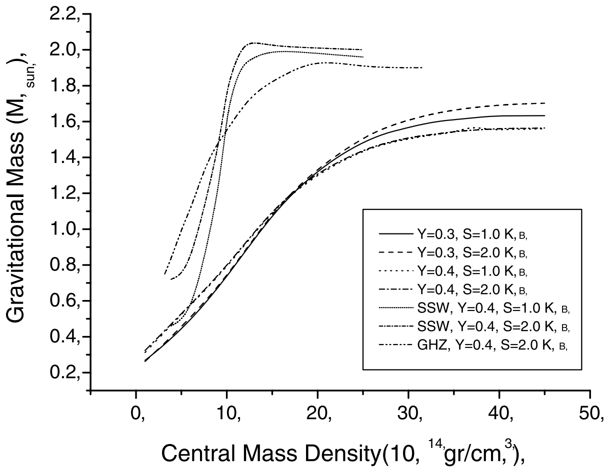



**Fig. 2** As Fig. 1, but for the gravitational mass ($M_\odot$) of protoneutron star versus central mass density ($10^{14}$ g/cm$^3$)

**Table 1** Our results for the maximum mass ($M_\odot$) of protoneutron star at different values of entropy ($s$) and lepton fraction ($Y$). The results of Gondek et al. [18] (GHZ) and Strobel et al. [19] (SSW) are also given for comparison

| | $s = 1.0k_B$ | $s = 2.0k_B$ |
|---|---|---|
| $Y = 0.4$ | 1.55 | 1.56 |
| $Y = 0.3$ | 1.63 | 1.70 |
| SSW, $Y = 0.4$ | 2.01 | 2.03 |
| GHZ, $Y = 0.4$ | | 1.91 |

**Table 2** Our results for the radius (km) of protoneutron star corresponding to the maximum mass at different entropies and lepton fractions

| | $s = 1.0k_B$ | $s = 2.0k_B$ |
|---|---|---|
| $Y = 0.4$ | 7.62 | 7.68 |
| $Y = 0.3$ | 8.13 | 8.35 |

Fig. 2 and Table 1, the results of Gondek et al. [18] and Strobel et al. [19] are also given for comparison. It is seen that these results are different from our calculations.

The radius of protoneutron star as a function of central mass density is shown in Fig. 3 for different entropies and lepton fractions. It is seen that as the central mass density increases, the radius decreases very rapidly and then reaches a nearly constant value. The radius of protoneutron star corresponding to the maximum mass for different entropies and lepton fractions is given in Table 2. It can be seen that the stiffer equation of state leads to the relatively higher radius.

The variations of temperature throughout the protoneutron star matter with respect to the radial coordinate ($r$) are presented in Figs. 4 and 5 for $s = 1k_B$ and $s = 2k_B$ with $Y = 0.4$ and $Y = 0.3$. We can see from the Figs. 4 and 5 that the temperature is decreasing slowly by increasing $r$, but it drops rapidly near the protoneutron star crust.

### 3.2 Beta-Stable Protoneutron Star

After complete deleptonization, neutrino trapped within the hot interior matter of neutron star do not affect the beta stability condition and therefore the lepton fraction is determined from the beta-equilibrium criteria [2–5]. In this stage, we have calculated the structure properties of protoneutron star for both isentropic and isothermal paths.

**Fig. 3** Our results for the radius (km) of protoneutron star versus central mass density ($10^{14}$ g/cm$^3$) at entropies $s = 1k_B$ and $s = 2k_B$ with lepton fractions $Y = 0.4$ and 0.3

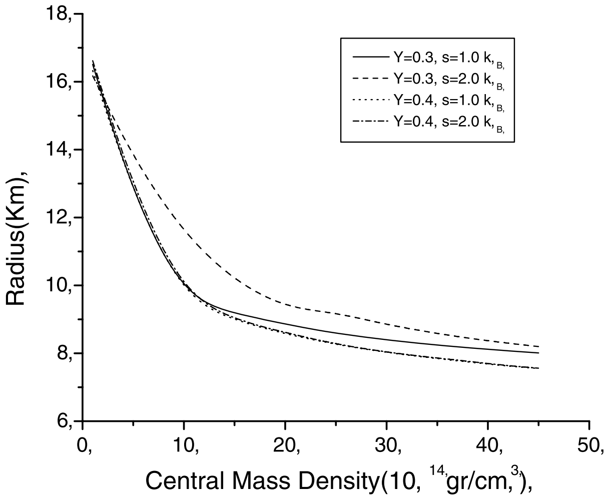
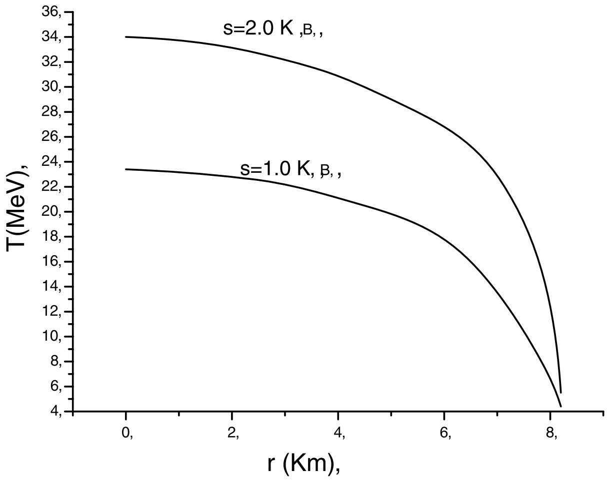



**Fig. 4** Our results for the temperature profile of protoneutron star at different entropies for $Y = 0.4$




### *3.2.1 Isentropic Paths*

For different values of entropy, our calculated equations of state of beta-stable protoneutron star are shown in Fig. 6. It is seen that the pressure of beta-stable protoneutron star matter increases by increasing entropy. In Fig. 6, the results of Strobel et al. [19] are also plotted for comparison. There is a compatible difference between our results and those of Strobel et al. [19].

Our calculated gravitational mass of beta-stable protoneutron star as a function of central mass density for $s = 1.0k_B$ and $s = 2.0k_B$ is presented in Fig. 7. From this figure, the increasing of gravitational mass by increasing both central mass density and entropy is seen. As it is shown in Fig. 7, for higher values of central mass density, the gravitational mass shows a limiting value (maximum mass). The maximum mass of beta-stable protoneutron star is given in Table 3 for different entropies. It is seen that the maximum mass decreases by

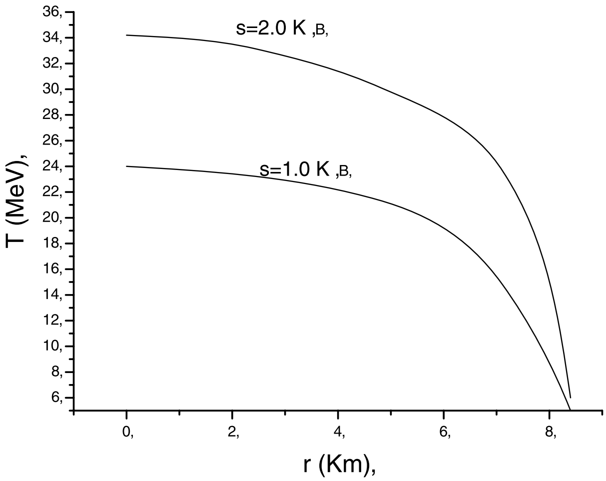



**Fig. 5** As Fig. 4, but for $Y = 0.3$

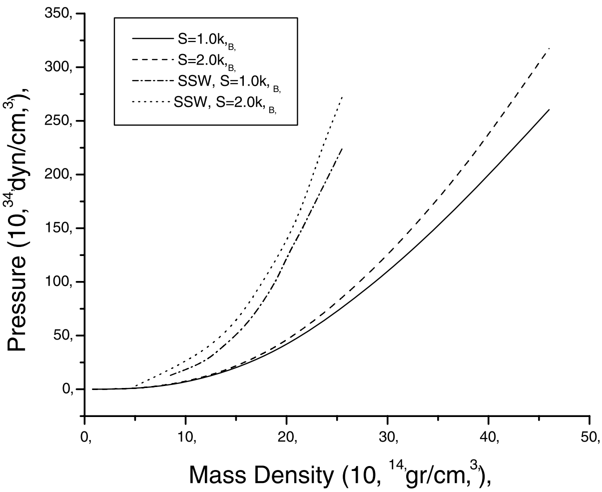



**Fig. 6** Our results for the pressure ($10^{34}$ dyn/cm$^2$) of beta-stable protoneutron star matter as a function of mass density ($10^{14}$ g/cm$^3$) at different entropies. The results of Strobel et al. [19] (SSW) are also given for comparison

decreasing the entropy. In Fig. 7 and Table 3, we have compared our results with the results of Strobel et al. [19]. It is seen that our results are different from those of Strobel et al. [19].

In Fig. 8, our results for the radius of beta-stable protoneutron star as a function of central mass density are plotted at different values of entropy. We see that the radius decreases very rapidly by increasing the central mass density and then exhibits a nearly constant value. The radius corresponding to the maximum mass of beta stable protoneutron star for $s = 1.0k_B$ and $s = 2.0k_B$ are also given in Table 3. This shows the decreasing of radius with respect to decreasing entropy.

The temperature of beta-stable protoneutron star matter as a function of radial coordinate ($r$) is shown in Fig. 9 for different entropies. The same as Figs. 4 and 5, we see that the temperature decreases slowly by increasing $r$, but drops rapidly near the beta-stable protoneutron star crust.

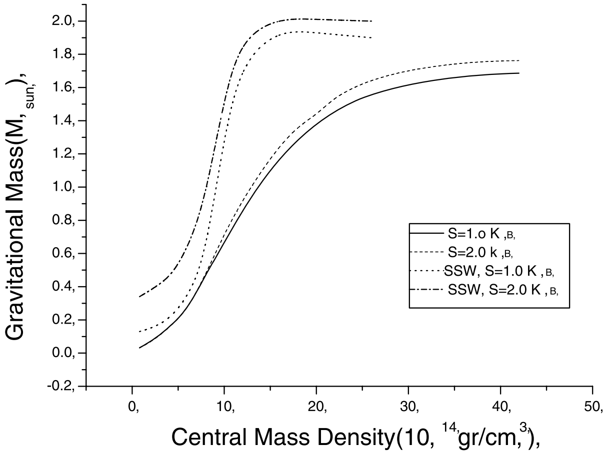



**Fig. 7** As Fig. 6, but for the gravitational mass ($M_\odot$) of beta-stable protoneutron star versus central mass density ($10^{14}$ g/cm$^3$)

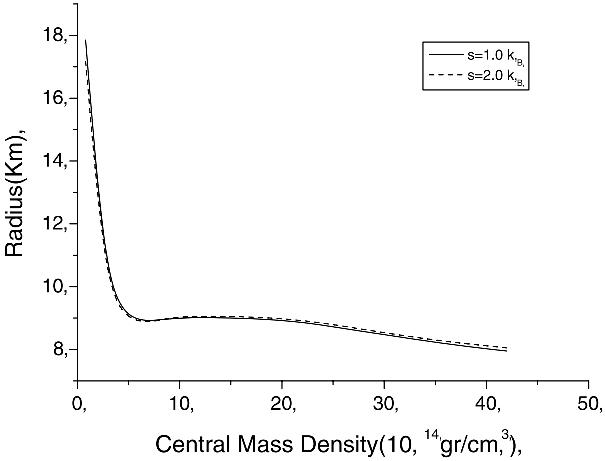



**Fig. 8** Our results for the radius (km) of beta-stable protoneutron star versus central mass density ($10^{14}$ g/cm$^3$) at different entropies

**Table 3** Our results for the maximum mass ($M_\odot$) of beta-stable protoneutron star and corresponding radius (km) at different entropies. The results of Strobel et al. [19] (SSW) for the maximum mass of beta-stable protoneutron star are also given for comparison

| | $s = 1.0k_B$ | $s = 2.0k_B$ |
|---|---|---|
| Mass | 1.68 | 1.76 |
| Radius | 8.05 | 8.15 |
| SSW | 1.98 | 2.03 |

### 3.2.2 Isothermal Paths

We are now able to do the above calculations for the protoneutron star in the beta-equilibrium case at different isothermal paths.

Our results for the equation of state, gravitational mass and radius of beta-stable protoneutron star at different values of temperature are presented in Figs. 10–12, respectively.

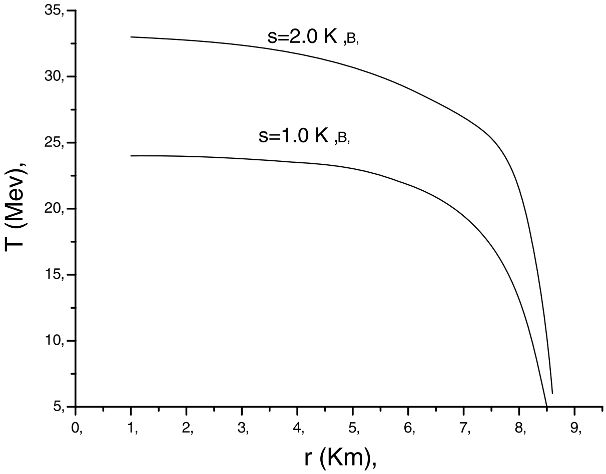



**Fig. 9** Our results for the temperature profile of beta-stable protoneutron star at different entropies

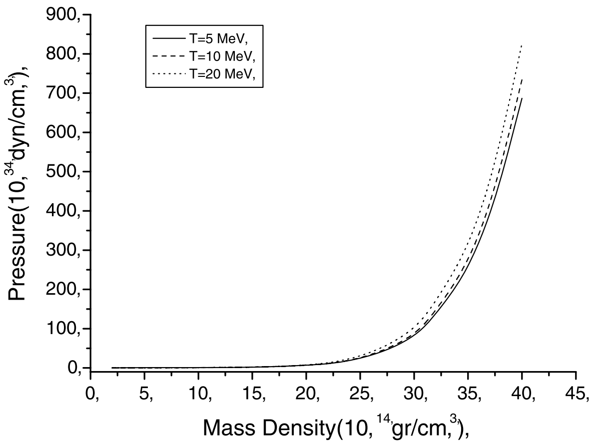



**Fig. 10** Our results for the pressure ($10^{34}$ dyn/cm$^2$) of beta-stable protoneutron star matter as a function of mass density ($10^{14}$ g/cm$^3$) at different temperatures

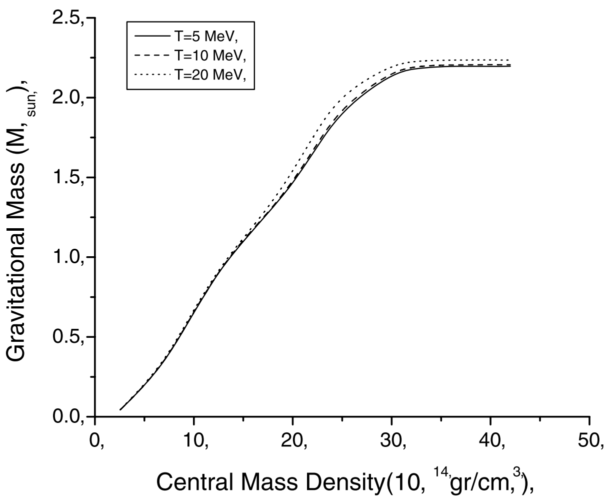



**Fig. 11** As Fig. 10, but for the gravitational mass ($M_{\odot}$) of beta-stable protoneutron star versus central mass density ($10^{14}$ g/cm$^3$)

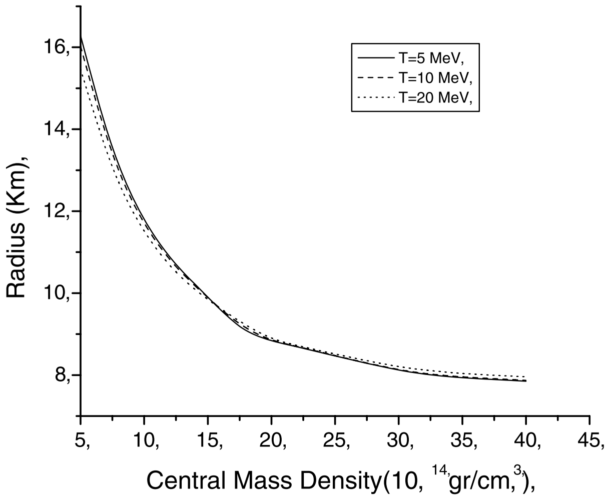



**Fig. 12** As Fig. 10, but for the radius (km) of beta-stable protoneutron star versus central mass density ($10^{14}$ g/cm$^3$)

**Table 4** Our results for the maximum mass ($M_\odot$) of beta-stable protoneutron star and corresponding radius (km) at different temperatures

| | $T = 5.0$ MeV | $T = 10.0$ MeV | $T = 20.0$ MeV |
|---|---|---|---|
| Mass | 2.19 | 2.20 | 2.23 |
| Radius | 7.85 | 7.88 | 8.01 |

At different temperatures, we have extracted the maximum mass and corresponding radius from these figures. Our results are given in Table 4. From this table, we can see that the maximum mass and radius of beta-stable protoneutron star decrease by decreasing the temperature.

## 4 Summary and Conclusion

The protoneutron stars which are lepton rich and hot objects are formed just after the supernova explosion. In this paper, we have integrated TOV equation to compute the maximum mass, radius and temperature profile for this object. As we have shown at a fixed value of lepton fraction, the mass and radius of protoneutron star decreases by decreasing entropy. The temperature is nearly constant in the core of protoneutron star and drops rapidly near its crust. The same properties of protoneutron star at beta equilibrium condition have been also calculated along the isentropic and isothermal paths. We have shown that the maximum mass and corresponding radius increase by increasing temperature.

**Acknowledgements** This work has been supported by Research Institute for Astronomy and Astrophysics of Maragha. We wish to thanks Shiraz University Research Council.

## References

1. Shapiro, S., Teukolsky, S.: Black Holes, White Dwarfs, and Neutron Stars. Wiley, New York (1983)
2. Pons, J.A., et al.: Astrophys. J. **513**, 780 (1999)
3. Burrows, A., Lattimer, J.M.: Astrophys. J. **307**, 178 (1986)

4. Bethe, H.A.: Rev. Mod. Phys. **62**, 801 (1990)
5. Burrows, A., Hayes, J., Fryxell, B.A.: Astrophys. J. **450**, 830 (1995)
6. Keil, W., Janka, H.Th.: Astron. Astrophys. **296**, 145 (1995)
7. Janka, H.Th.: astro-ph/0402200 (2004)
8. Strobel, K., Weigel, M.K.: Astron. Astrophys. **367**, 582 (2001)
9. Gondek, D., Haensel, P., Zdunik, J.L.: astro-ph/0012541 (2000)
10. Bordbar, G.H.: Int. J. Theor. Phys. **41**, 309 (2002)
11. Bordbar, G.H., Riazi, N.: Int. J. Theor. Phys. **40**, 1671 (2001)
12. Bordbar, G.H., Riazi, N.: Int. J. Theor. Phys. Group Theory Nonlinear Opt. **7**, 73 (2001)
13. Bordbar, G.H., Riazi, N.: Astrophys. Space Sci. **282**, 563 (2002)
14. Bordbar, G.H., Hayati, M.: Int. J. Mod. Phys. A **21**, 1555 (2006)
15. Wiringa, R.B., Stoks, V., Schiavilla, R.: Phys. Rev. C **51**, 38 (1995)
16. Bordbar, G.H., Modarres, M.: J. Phys. G Nucl. Part. Phys. **23**, 1631 (1997)
17. Bordbar, G.H., Modarres, M.: Phys. Rev. C **57**, 714 (1998)
18. Gondek, D., Haensel, P., Zdunik, J.L.: Astron. Astrophys. **325**, 217 (1997)
19. Strobel, K., Schaab, Ch., Weigel, M.K.: Astron. Astrophys. **350**, 497 (1999)